Eastern Michigan University
Division of Academic Affairs

Request for Inclusion of a Course in the General Education Program: Education for Participation in the Global Community

Department/School: Mathematics College: Arts and Sciences

Department Contact: Gisela Ahlbrandt Contact Phone: 487-1444
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1. **Subject Code, Number, and Title: MATH 120 Calculus I**

2. Credit Hours: Four
3. Catalog description: Calculus of functions of a single variable; differential calculus, including limits, derivatives, techniques of differentiation, the Mean Value Theorem and applications of differentiation to graphing, optimization and rates. Integral calculus, including indefinite integrals, the definite integral, the Fundamental Theorem of Integral Calculus, and applications of integration to area and volume.

4. This course is (check one):

☑ an existing course with no revisions (need not go through the input system)

☐ an existing course with revisions (attach this form to Request for Course Revision form)

☐ a new course (attach this form to Request for New Course form)

Check the General Education requirement this course is intended to meet. If the course is to be proposed for more than one requirement, submit a separate form for each one.

☐ **Effective Communication**

☑ **Quantitative Reasoning (*QR designation*)**

☐ **Writing Intensive (*WI designation*)**

☐ **Perspectives on a Diverse World**

- ☐ Global Awareness
- ☐ U.S. Diversity

☐ **Knowledge of the Disciplines**

- ☐ Arts
- ☐ Humanities
- ☐ Science
- ☐ Social Science

☐ **Learning Beyond the Classroom (*LBC designation*)**

6. Rationale. Provide a concise, clear, jargon-free explanation of why this is a General Education course and how it fits into this specific requirement. This rationale should appear on the general course syllabus provided here and should be included in specific course syllabi given to students.

MATH 120 is an introductory four credit course in calculus. Students in this course will develop the mathematical skills associated with the core topics of limits, derivatives and integration, and learn the wider context for these skills within the mathematical sciences.

In a unified fashion, the course makes the case for using functions to model physical phenomena and simultaneously teaches methods to analyze these functions in a meaningful way. Applications of calculus abound in the physical and life sciences and, increasingly, in social sciences like economics as well. It is the theoretical engine that is used in these client disciplines when it comes time to reason in a quantitative way. For these reasons, MATH 120 will count for the **Quantitative Reasoning** requirement in the General Education program Education for Participation in the Global Community.

7. Clearly and concisely explain how this course meets each of the General Education outcomes for the requirement checked in number five (all outcomes should be addressed). To do this, (a) list the General Education outcomes for the requirement and explain how the course meets each outcome; and (b) explain, in general terms, the method(s) of evaluation to be used in the course and how these methods assess the degree to which students have met the General Education outcomes for this requirement.

Outcomes for quantitative reasoning:

Students will learn to solve real-life problems using a mathematical modeling process. They will learn to…

1. Identify an appropriate model.

a. Students will explore various application domains where calculus provides the right model for quantitative analysis. The basic concept of derivative provides a way of discussing instantaneous rate of change, which is used in the hard sciences and increasingly in the social sciences (velocity, acceleration, marginal profit and cost, reaction rates, *etc*). First semester calculus is well stocked with examples where such models are analyzed in order to provide answers to important applied questions—related rates and optimization problems, for example.

As a specific example of the latter: the problem is to find the dimensions of a rectangular box with no lid and fixed volume that has minimal surface area. The student has to first come up with a function model for the surface area, by specifying variables and encoding assumptions that are in this case given. Only when that is done can the students apply techniques of calculus and solve the problem.

b. Assessment of this outcome can be via questions on quizzes and tests. In the example just given, the problem itself is not to set up a model, but the solution of the given problem using calculus requires the student to set up an appropriate model.

2. Identify and discuss assumptions.

a. In the presentation of any quantitative real-world situation, the class will discuss the assumptions necessary to proceed to an appropriate mathematical model. For example, consider one important application of calculus to physics: uniform acceleration due to gravity. The starting point in the analysis is the assumption that bodies fall under the influence of gravity with constant acceleration. It is important to point out that this assumption only yields accurate results when the body in question is close to the surface of the Earth. When this underlying assumption is not in place, as for example with bodies in orbit, different models must be used. Discussions of such issues are typical in any calculus course when teaching applications.

b. This outcome can be assessed via direct questions on quizzes and tests, where a situation is presented and the student discusses the assumptions he/she might make in selecting an appropriate model. In a problem involving motion under gravity over small distances, the essential assumptions are that the force of gravity on the body is constant, and that gravity is the only force acting on the body (e.g., there is no air resistance). In a problem on population growth, as assumption that leads to an informative model is that the rate of growth is proportional to the population size.

3. Collect or generate appropriate data.

a. Many of the fundamental questions in calculus are driven by numerical evidence. For example, to understand the concept of limit it is crucial to explore numerical data in many concrete situations. This is also true when students meet two other central calculus constructs: the derivative and the integral. As an example, when students are learning the concept of derivative, they might be given a function model for the height of a body moving under a constant gravitational force, without air resistance. Using this function model, they can generate data in order to calculate average velocities over various time intervals. [The point of the exercise is then to see that if the average velocities are taken over shorter and shorter time intervals, there appears to be a limiting value to which those average velocities are approaching.]

b. The above example is likely to be given on an extended homework problem. A similar type of example would be when students are learning the concept of the definite integral, and are asked to produce lower and upper bounds for a net change in the volume of water in a tank, given that values are known for the rate of flow into the tank at different times. Problems like this are likely to given for homework rather than on a quiz or test because the numerical computation involved can be somewhat time-consuming.

4. Analyze a situation using arithmetic, geometric, algebraic, and probabilistic or statistical methods.

a. Every major calculus concept should be understood in three different ways: numerically, geometrically and algebraically. The derivative provides an illustrative example. This notion is equivalent to the idea of instantaneous rate of change that is exploited in many application domains. It can be approximated numerically by a limit process. The derivative is also at work in the geometric concept of tangent line. Finally, the precise definition of the derivative is a formula whose computation involves algebraic manipulation. The student is exposed to all three approaches and the interplay between them. Integrals, introduced at the end of the course, are also taught this way. In addition, they can also be computed using a probabilistic method called the Monte Carlo method.

b. A staple of calculus quizzes and tests is various types of "word problem," and of course many are given for homework as well. Just about any calculus word problem involves either arithmetic and algebraic methods, or else graphical methods, and often both; and many also involve geometric methods such as the Pythagorean Theorem, similar triangles, or various trigonometric relationships.

5. Estimate answers.

a. Approximation processes power both the idea of derivative and of the integral. Both concepts are defined as the limiting value of the appropriate estimation process. Students will explore these approximation schemes and how they converge to the exact answer in many different contexts.

b. As mentioned above, students may be asked to use various average velocities to estimate an instantaneous velocity, or to estimate a net change in volume by calculating upper and lower bounds based on given values for the rate of flow. As another example, students may be asked to estimate a value for a quantity such as using approximation methods based on the derivative.

6. Propose and evaluate solutions.

a. The calculus curriculum includes topics that require the development of mathematical models for physical situations (related rates, optimization). The student must understand a verbal description of a problem and then encode the problem within a mathematical framework in order to arrive at a solution. Sometimes there is a formal method for checking some aspect of a solution, but even if not the student should evaluate the solution for its reasonableness given the original problem situation.

b. Students will be asked on homeworks, quizzes and tests to solve problems which require setting up a function model and then applying a calculus technique to solve the problem. See the rectangular box example in Outcome 1a. In this same problem, once a solution has been found, the student should use a related technique to check that the surface area they have arrived at is indeed a minimum.

7. Predict outcomes in other situations based on what they have learned from their analysis.

a. The practice of mathematics as an analytic discipline relies heavily on inductive reasoning. General patterns are usually noticed by examining many different special cases. This 'building-up" of skill and knowledge is evident in many situations within calculus. One example of this is the derivation of some of the general formulas for derivatives. The power rule can be effectively anticipated by the student by working out several special cases. As another example: the problem is to find the dimensions of a rectangular enclosure that can be built along an existing wall using a given amount of fencing material for the other three sides, so as to have maximum area. If students are given this problem with a specific length of fencing material, they get a solution that is twice as long as it is wide. They can easily guess that this same relationship will hold true no matter what length of fencing material is supplied, and they can then go on to prove their conjecture by using a parameter in place of the given length.

b. Students will be given problems throughout the course that foster the development of inductive reasoning. Such problems would likely be given as homework problems, but simple examples can be given on quizzes and tests.

8. Understand and communicate quantitative relationships using symbols, equations, graphs, and tables.

a. All of the topics covered in the class will involve symbols, equations, graphs and tables. Students will learn to use symbols effectively, to understand and be able to effectively use equations, and to use and understand graphs and tables. Calculus I is part of a long road that students take to develop these skills. They have been learning to manipulate symbols, to work with equations, and to interpret graphs and tables of numerical data since at least 6^{th} grade. All of these skills take a lot of time to develop, and all are essential to gaining a good understanding of calculus. Students develop these skills all semester long in every topic that is covered in the course. Mostly it is a matter of repetition and continued exposure. Students find that skills they learned for the test and then forgot back in Algebra 2 are essential on a day-by-day basis, and they learn them without even trying to, because they have become survival skills.

b. There's no way a student can pass Calculus I without being able to do these things. They are assessed on every homework assignment, quiz and test, in every single question.

9. Share their findings in oral and written reports using appropriate mathematical language.

a. & b. Part of the mathematical development of a student is learning to communicate effectively in a mathematical setting. In high school, students have often been allowed to work a problem in any haphazard way as long as they put a box around their answer or write it on the right part of the quiz sheet. Instructors in Calculus I are part of a process of getting students to learn to write complete solutions to problems, connecting steps together and showing the logical flow of a solution to a problem. Many instructors use group work in Calculus I, and have the opportunity to assess how well students communicate with each other both in written and in oral form. Instructors may also assign homework problems that require a full write-up (as opposed to an answer with a box around it). Students may be graded not just on the correctness of the algebra or of the numerical answer, but also on the correct use of symbols and on the placing of the answer in one of more sentences which state the answer clearly and at the same time place it in context.

10. Write summaries to explain how they reached their conclusions.

a. & b. Students will learn to summarize the work they have done in analyzing a given situation. Students might be asked to reflect on a problem solving task they have accomplished and explain to each other or the instructor how they went about solving the problem. In some problems there may be an amount of numerical work (see Outcome 3) from which the student is expected to draw an inference, and the student will need to summarize the evidence in order to support whatever inference he/she might make.

11. Draw inferences from a model.

a. In most applied problems in calculus, the model is set up in order to solve a specific problem, so that part of the problem method is to draw an inference from the model. For example, once the function model for the rectangular box problem (see Outcome 1a) has been set up, calculus techniques are applied, and a solution is reached. The desired solution is then an inference drawn from the model. In the case of projectile motion, once the assumptions have been made and the calculus has been done, the student ends up with a number of inferences drawn from the model—how high the projectile went, how long it was in the air, how fast it was going when it hit the ground, etc.

b. This outcome will be evaluated in word problems on quizzes, tests and homework.

12. Discuss the limitations of the model.

a. An essential part of the discussion of any model will be an analysis of the assumptions made in constructing it, and its limitations as a model for the given situation. Projectile motion provides a good example, but it is just one of many. In projectile motion it is standard to assume that the force of gravity on the body is constant, and that there are no other forces present. Once some answers have been found (see Outcome 11), it is interesting to ask students how they would vary in the presence of air resistance. Will the projectile go higher or not as high? Will it go farther or not as far? Will it stay in the air longer or hit the ground sooner? A fun example to discuss is the penny dropped off the top of the Empire State Building. How fast will it be going when it hits the ground? Then go to the web and search for the terminal velocity of a falling penny (or watch the relevant Mythbusters episode).

b. This outcome will be evaluated in word problems on quizzes, tests and homework.

8. Attach a syllabus (1-inch margins and 10-12 pt. font). The syllabus must include the rationale from #6 above and clearly reflect the outcomes and methods detailed in #7 above.

MATH 120 – Mathematical Reasoning

Syllabus

{Note: This is a generic syllabus. Some parts will be common to all sections. Others will vary by instructor, and are provided here as examples, for completeness. These parts are enclosed in square brackets [ ]. Other explanatory notes that would not be part of an actual syllabus, such as this note, are enclosed in braces {} .}

<u>Course Catalog Description</u>
Calculus of functions of a single variable; differential calculus, including limits, derivatives, techniques of differentiation, the Mean Value Theorem and applications of differentiation to graphing, optimization and rates. Integral calculus, including indefinite integrals, the definite integral, the Fundamental Theorem of Integral Calculus, and applications of integration to area and volume. Knowledge of trigonometry is assumed.
Prerequisite: Placement or at least a C in Math 105 and MATH 107, or in Math 112 or in Math 210 and MATH 107.

<u>Goal of the Course</u>
MATH 120 is the first part of a two semester sequence in Calculus. The objective is to develop fluency in the mathematical skills associated with the subject as well as an understanding of its role within the sciences.

<u>Course Objectives</u>
Upon completing the course, students should:

have a good understanding of the core concepts of calculus: limits, differentiation, integration,

be able to use their understanding in solving a wide variety of problems that require calculus, and

have a good understanding of how the language of calculus is useful within various disciplines like physics, chemistry, biology and economics.

<u>General Education</u>
MATH 120 is an introductory four credit course in calculus. Students in this course will develop the mathematical skills associated with the core topics of limits, derivatives and integration, and learn the wider context for these skills within the mathematical sciences.

In a unified fashion, the course makes the case for using functions to model physical phenomena and simultaneously teaches methods to analyze these functions in a meaningful way. Applications of calculus abound in the physical and life sciences and, increasingly, in social sciences like economics as well. It is the theoretical engine that is used in these client disciplines when it comes time to reason in a quantitative way. For these reasons, MATH 120 will count for the **Quantitative Reasoning** requirement in the General Education program Education for Participation in the Global Community.

<u>Text:</u> {One of two texts is typically used:
Calculus, Single Variable, 3rd Edition *by Deborah Hughes-Hallett, Andrew M. Gleason, et. al., John Wiley and Sons, 2003.*
Calculus, Early Transendentals, ***5th Edition,*** *by James Stewart, Brooks Cole, 2005.*
}

<u>Calculators</u>
You are expected to bring a graphing calculator to class. You may need to use calculators on assignments, quizzes, and tests.

Assessment {Policy depends on individual instructor}
[We will use the following grading scale:

A	90 - 100
B	80 - 89
C	70 - 79
D	60-69
E	Below 60

Your grade will be computed from a weighted average, with the following components:

Homework, In-Class Work and Projects	40%
Quizzes	10%
Tests (3)	30%
Final	20%]

Attendance & Class Participation
{Policy depends on individual instructor}

Homework, Quizzes, and In-Class Work
{Policy depends on individual instructor}

Tests
{Policy depends on individual instructor}

Written work
Written work must be presented in a format commensurate with that expected in any other college class. When appropriate to do so (in applied problems), you must write in complete sentences, with correct spelling, grammar and punctuation.

Makeup Policy
{Policy depends on individual instructor}

Extra Help
[You are always welcome to meet with me if you are having difficulties with course material. In addition, there is a math-tutoring center where you can get individual help in Room 220 of Pray-Harrold. The tutoring center is open 5 days a week. You also can drop by the Math Den in 501 Pray-Harrold, which has a library of basic math books and a place to study with other math students.]

Special Needs
If you have learning disability or other physical impairment that may affect your ability to do the work in this course, please let me know as soon as possible so that we can make appropriate arrangements. See the link at www.math.emich.edu/access.services.html for information about support services. In addition, international students should check the information at www.math.emich.edu/SEVIS.html concerning registration.

Academic Honesty
[I expect all students to abide by the University's code of conduct, and in particular to abide by rules concerning academic honesty. In order to assess how the class is going and what you have learned, I need to see your own work: your own words and the details of your own computations. You may work with other students or math tutors on your assignments, but you must do an independent write-up. I will give failing grades for academic dishonesty.]

Course Outline
{ A typical course outline might be as follows:
1. Limits
    - Limit of a Function
    - Calculating Limits using Limit Laws
    - Continuity
    - Tangents, Velocities and Other Rates of Change
2. Derivatives
    - Derivative of a Function
    - Derivatives of Polynomials and Exponential Functions
    - Rules for Differentiation (including the product, power and chain rules)
    - Derivatives of Trigonometric and Logarithmic Functions
    - Rates of Change in the Natural and Social Sciences
    - Related Rates
3. Applications of Derivatives
    - Extreme Values
    - The Mean Value Theorem and Consequences
    - Curve Sketching
    - Optimization Problems
4. Integration
    - Areas
    - The Definite Integral
    - The Fundamental Theorem of Calculus
    - Indefinite Integrals
    - Substitution

}

Please submit all materials in electronic form.

Action of the Department/College

1. Department

Vote of department faculty: For ___14_____ Against ___0______ Abstentions ___1______

Department Head: Signed by Bette Warren Date: March 20, 2007

2. College

College Dean Date

Action of General Education Advisory Committee

Vote of General Education Committee: For ________ Against ________ Abstentions ________

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Chairperson, General Education Advisory Committee Date

Approval

Associate Vice-President for Undergraduate Studies and Curriculum Date