Eastern Michigan University
Division of Academic Affairs

Request for Inclusion of a Course in the General Education Program: Education for Participation in the Global Community

Department/School: Mathematics **College:** Arts and Sciences

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1. **Subject Code, Number, and Title: MATH 119 Applied Calculus**
2. Credit Hours: Three
3. Catalog description:

Introduction to the concepts and applications of differential and integral calculus: behavior and properties of algebraic, logarithmic and exponential functions, derivatives and rates of change, optimization and definite integral as accumulation. Emphasis on problem setup, interpretation and applications.

4. This course is (check one):

- ☑ an existing course with no revisions (need not go through the input system)
- ☐ an existing course with revisions (attach this form to Request for Course Revision form)
- ☐ a new course (attach this form to Request for New Course form)

5. Check the General Education requirement this course is intended to meet. If the course is to be proposed for more than one requirement, submit a separate form for each one.

- ☐ **Effective Communication**
- ☑ **Quantitative Reasoning (*QR designation*)**
- ☐ **Writing Intensive (*WI designation*)**
- ☐ **Perspectives on a Diverse World**
  - ☐ Global Awareness
  - ☐ U.S. Diversity
- ☐ **Knowledge of the Disciplines**
  - ☐ Arts
  - ☐ Humanities
  - ☐ Science
  - ☐ Social Science

☐ **Learning Beyond the Classroom (*LBC designation*)**

6. Rationale. Provide a concise, clear, jargon-free explanation of why this is a General Education course and how it fits into this specific requirement. This rationale should appear on the general course syllabus provided here and should be included in specific course syllabi given to students.

MTH 119 is an introductory three credit course in applied calculus. Students in this course will develop the mathematical skills associated with the core topics of derivatives and integration, and learn to apply these skills within economics, finance, and the life and social sciences.

In a unified fashion, MATH 119 makes the case for using functions to model phenomena in the social sciences, and simultaneously teaches methods to analyze these functions in a meaningful way. Applications of calculus abound in the social sciences and economics. For these reasons, MATH 119 will count for the **Quantitative Reasoning** requirement in the General Education program Education for Participation in the Global Community.

7. Clearly and concisely explain how this course meets each of the General Education outcomes for the requirement checked in number five (all outcomes should be addressed). To do this, (a) list the General Education outcomes for the requirement and explain how the course meets each outcome; and (b) explain, in general terms, the method(s) of evaluation to be used in the course and how these methods assess the degree to which students have met the General Education outcomes for this requirement.

Outcomes for quantitative reasoning:

Students will learn to solve real-life problems using a mathematical modeling process. They will learn to…

1. Identify an appropriate model.

a. Students will explore various application domains where calculus provides the right model for quantitative analysis. The basic concept of derivative provides a way of discussing instantaneous rate of change, which is used in the social sciences (marginal profit and cost, rates of growth, etc). Applied calculus is well stocked with examples where such models are analyzed in order to provide answers to important applied questions.

The following optimization problem is an example: the problem is to find the dimensions of a rectangular box with no lid and fixed volume that has minimal surface area. The student has to first come up with a function model for the surface area, by specifying variables and encoding assumptions that are in this case given. Only when that is done can the students apply techniques of calculus and solve the problem.

b. Assessment of this outcome can be via questions on quizzes and tests. In the example just given, the problem itself is not to set up a model, but the solution of the given problem using calculus requires the student to set up an appropriate model. A specific example: Acme Packaging is asked to design a box which has a volume of 250 cubic inches. The box is to have a square base and be open at the top. The box should be constructed out of the least possible amount of material. In this problem, students have to identify appropriate variables and set up a function model for the surface area of the box; they can then use calculus techniques to find the minimum.

2. Identify and discuss assumptions.

a. In the presentation of any quantitative real-world situation, students will discuss the assumptions necessary to proceed to an appropriate mathematical model. For example, consider one important application of calculus to the life sciences: exponential growth. In using an exponential model to describe the growth of a population for a certain time frame, one has to assume that the percentage growth rate is a constant. Discussions of such issues are typical in any calculus course when teaching applications.

b. This outcome can be assessed via direct questions on quizzes and tests, where a situation is presented and the student discusses the assumptions he/she might make in selecting an appropriate model. In the last examples, students could be asked as part of the problem to identify and discuss assumptions. Another example: A population of 5,000 increases to 8,000 over 3 years. Give two different assumptions you can make about the rate of population growth, which lead to different models for the populations size. Under what circumstances might one model be preferable to the other?

3. Collect or generate appropriate data.

a. There is a multitude of data available on the Internet, which students can use to construct models for this class. For example they can find the population of the United States for several years and then use their data to fit an exponential model. In a similar way they can find the historical rates for the consumer price index from the website of the bureau of labor statistics and then use these data to set up an exponential model to find the average rate of inflation over the last 30 years.

b. The above examples are likely to be given on a homework problem. Another example: a. Use the RAND function on your calculator to generate 10 random points in the xy-plane in the square $0 \le x \le 1$, $0 \le y \le 1$. Use these points to estimate the integral $\int_0^1 x^2 dx$. b. Now use the program Monte Carlo to estimate this integral using 1000 points. Compare both answers with the exact value. In the (a) part of this problem, students use the random number generator in their calculator (pseudo random numbers really, but who is checking?) to generate data which can then be used to estimate an integral.

4. Analyze a situation using arithmetic, geometric, algebraic, and probabilistic or statistical methods.

a. Every major calculus concept should be understood in three different ways: numerically, geometrically and algebraically. The derivative provides one illustrative example. This notion is equivalent to the idea of instantaneous rate of change that is exploited in many application domains. It can be approximated numerically by a limit process. The derivative is also at work in the geometric concept of tangent line. Finally, the precise definition of the derivative is a formula whose computation involves algebraic manipulation. The student is exposed to all three approaches and the interplay between them. In situations requiring probabilistic methods probabilities can be computed as areas under a probability density function using integration.

b. Nearly every problem on a quiz, test or homework assignment will require these skills.

5. Estimate answers.

a. Estimation of ballpark answers to problems is an essential part of solving problems in a Calculus class. Since students use calculators it is important that they are able to give an order of magnitude estimate for the solution. In addition approximation processes power both the idea of derivative and integral. Both concepts are defined as the limiting value of the appropriate estimation process. Students can also use a curve fitting approach to estimation. As in the previously mentioned example, they can use an exponential model for the population of the United States to estimate the annual percentage rate of population growth for the last 10 years.

b. This can be done in quizzes, tests and homework assignments. An example: A population of 5000 grows to 8000 in 3 years. Make an estimate of the population 3 years later, and explain how you got it. What assumption is your estimate based on?

6. Propose and evaluate solutions.

a. The calculus curriculum includes topics that require the development of mathematical models for economical and financial situations. The student must understand a verbal description of a problem and then encode the problem within a mathematical framework in order to arrive at a solution. For example the students may set up a model to compare the present value of an income stream, like an installment payment on a lottery win, with a lump-sum payment. As the students solve problems in this setting they will propose solutions and evaluate them for their reasonableness given the original problem situation. In this example they might consider other factors, like tax considerations, inflation, the possibility to invest in an upstart company or financial security in old age, as they evaluate their mathematical solution.

b. Students will be asked on homework, quizzes and tests to solve problems that require setting up a function model and then applying a calculus technique to solve the problem. They then need to evaluate their solution in the light of the original problem situation.

7. Predict outcomes in other situations based on what they have learned from their analysis.

a. The practice of mathematics as an analytic discipline relies heavily on inductive reasoning. General patterns are usually noticed by examining many different special cases. This 'building-up" of skill and knowledge is evident in many situations within calculus. One example of this is the derivation of some of the general formulas for derivatives. The power rule can be effectively anticipated by the student by working out several special cases. As another example: the problem is to find the dimensions of a rectangular enclosure that can be built along an existing wall using a given amount of fencing material for the other three sides, so as to have maximum area. If students are given this problem with a specific length of fencing material, they get a solution that is twice as long as it is wide. They can easily guess that this same relationship will hold true no matter what length of fencing material is supplied, and they can then go on to prove their conjecture by using a parameter in place of the given length.

b. Students will be given problems throughout the course that foster the development of inductive reasoning. Such problems would likely be given as homework problems, but simple examples can be given on quizzes and tests. An example: Find the area of the rectangle with perimeter 32" which has the least possible area. What do you notice? Can you generalize this observation? In this problem, a standard procedure leads to the answer that the rectangle is 8” by 8”. Students should note that this is a square, and be able to generalize this to a general statement about rectangle with fixed perimeter and smallest area (the so-called isoperimetric problem for rectangles). [For the general isoperimetric problem where all possible shapes are allowed, the solution is a circle. It is common sense once you think about it, but try proving it!]

8. Understand and communicate quantitative relationships using symbols, equations, graphs, and tables.

a. All of the topics covered in the class will involve symbols, equations, graphs and tables. Students will learn to use symbols effectively, to understand and be able to effectively use equations, and to use and understand graphs and tables. Applied Calculus is part of a long road that students take to develop these skills. They have been learning to manipulate symbols, to work with equations, and to interpret graphs and tables of numerical data since at least 6^{th} grade. All of these skills take a lot of time to develop, and all are essential to gaining a good understanding of calculus. Students develop these skills all semester long in every topic that is covered in the course. Mostly it is a matter of repetition and continued exposure. Students find that skills they learned for the test and then forgot back in Algebra 2 are essential on a day-by-day basis, and they learn them without even trying to, because they have become survival skills.

b. Again, nearly every problem on a quiz, test or homework assignment will require these skills.

9. Share their findings in oral and written reports using appropriate mathematical language.

a. Applied Calculus provides many opportunities to investigate a real life situation and investigate it in terms of the mathematics learned in the course. The Gini index, for example, is a measure used by economists to compare the degree of equitable distribution of wealth of countries. Students can investigate the details of the quite subtle economic definition and rephrase it in terms of the area between two curves, i.e. a definite integral, one of the central topics in any Calculus course. Students can then share their insights in class discussions or submit a project they developed with the help of guided investigations and their own research.

b. Extended projects will most likely be assigned as homework. The Gini index mentioned above would make a good subject for a project—it is very interesting to compare the Gini index for the US in 2005 with that from 1975. In addition essay questions on a test might assess the understanding gained from a project.

10. Write summaries to explain how they reached their conclusions.

a. Students will learn to summarize the work they have done in analyzing a given situation. Students might be asked to reflect on a problem-solving task they have accomplished and explain to each other or the instructor how they went about solving the problem. In writing up their project as described in item 9, students will need to give explanations on how they proceeded with the task.

b. This can be assessed in tests and quizzes as well as in extended homework assignments.

11. Draw inferences from a model.

a. Continuing with the example of the Gini index, students will learn that a large area between the so-called Lorenz curve and the diagonal is a measure of the inequality in a population. They can then compare Lorenz curves for different years for the US population and discover that the degree of inequality has increased over the last 30 years. Mathematical models used in an applied calculus class will often shed light on the real world situation, that's why the model was developed in the first place. Students will learn to discover what a model can imply about the situation.

b. This outcome will be evaluated in word problems on quizzes, tests and homework. An example: A population of 5,000 grows to 8,000 in 3 years. If we assume an exponential model for population growth, when will the population reach 10,000? In fact, it actually takes 3 more years to reach 10,000. What does this tell you about the population growth over this period? In this problem, students see that the prediction of the exponential model is not close to what we are told really happens. They can infer that the 17% growth rate of the first 3 years has dropped substantially, so something other than natural population increase has been at work here.

12. Discuss the limitations of the model.

a. An essential part of the discussion of any model will be an analysis of the assumptions made in constructing it, and its limitations as a model for the given situation. In working with an exponential model to describe the growth of the world population for example it is imperative that students are aware that unbounded exponential growth is not realistic for any material quantities. After some time the population will exceed the carrying capacity of the earth and the growth will have to cease. An exponential model cannot be used to describe this phase of declining rates of growth. Students will learn that an exponential model is only appropriate for a limited time span to describe actual populations.

b. This outcome will be evaluated in word problems on quizzes, tests and homework. As an example: Write a model for the size of a population which starts at 5,000 and grows to 8,000 in 3 years, assuming the growth is exponential. Explain why this model should not be used to predict the population in 100 years time. [If you try it, you will see that the population grows to 30 billion!]

8. Attach a syllabus (1-inch margins and 10-12 pt. font). The syllabus must include the rationale from #6 above and clearly reflect the outcomes and methods detailed in #7 above.

MATH 119 – Applied Calculus

Syllabus

{Note: This is a generic syllabus. Some parts will be common to all sections. Others will vary by instructor, and are provided here as examples, for completeness. These parts are enclosed in square brackets [ ]. Other explanatory notes that would not be part of an actual syllabus, such as this note, are enclosed in braces {} .}

Course Catalog Description

Calculus of functions of a single variable; differential calculus, including limits, derivatives, techniques of differentiation, the Mean Value Theorem and applications of differentiation to graphing, optimization and rates. Integral calculus, including indefinite integrals, the definite integral, the Fundamental Theorem of Integral Calculus, and applications of integration to area and volume. Knowledge of trigonometry is assumed.

Prerequisite: Placement or at least a C in Math 104 (B or better strongly recommended) or Math 105 or Math 210.

Goal of the Course

MATH 119 is an applied calculus course. The objective is to develop fluency in the mathematical skills associated with the subject as well as an understanding of its role within economics, finance, and the life and social sciences.

Course Objectives

Upon completing the course, students should:

- have a good understanding of the core concepts of calculus: limits, differentiation, integration
- be able to use their understanding in solving a wide variety of problems that require calculus, and
- have a good understanding of how the language of calculus is useful within various disciplines like physics, chemistry, biology and economics.

General Education

MATH 119 is an introductory three credit course in applied calculus. Students in this course will develop the mathematical skills associated with the core topics of derivatives and integration, and learn to apply these skills within economics, finance, and life and social sciences.

In a unified fashion, MATH 119 makes the case for using functions to model phenomena in the social sciences, and simultaneously teaches methods to analyze these functions in a meaningful way. Applications of calculus abound in the social sciences and economics. For these reasons, MATH 119 will count for the **Quantitative Reasoning** requirement in the General Education program Education for Participation in the Global Community.

Text: {The text which is currently used:
Applied Calculus, 3rd Edition *by Deborah Hughes-Hallett, Andrew M. Gleason, et. al., John Wiley and Sons, 2006.*}

Calculators
You are expected to bring a graphing calculator to class. You may need to use calculators on assignments, quizzes, and tests.

Assessment {Policy depends on individual instructor}
[We will use the following grading scale:

A	90 - 100
B	80 - 89
C	70 - 79
D	60-69
E	Below 60

Your grade will be computed from a weighted average, with the following components:

Homework, In-Class Work and Projects	40%
Quizzes	10%
Tests (3)	30%
Final	20%]

Attendance & Class Participation
{Policy depends on individual instructor}

Homework, Quizzes, and In-Class Work
{Policy depends on individual instructor}

Tests
{Policy depends on individual instructor}

Written work
Written work must be presented in a format commensurate with that expected in any other college class. When appropriate to do so (in applied problems), you must write in complete sentences, with correct spelling, grammar and punctuation.

Makeup Policy
{Policy depends on individual instructor}

Extra Help
[You are always welcome to meet with me if you are having difficulties with course material. In addition, there is a math-tutoring center where you can get individual help in Room 220 of Pray-Harrold. The tutoring center is open 6 days a week. You also can drop by the Math Den in 501 Pray-Harrold, which has a library of basic math books and a place to study with other math students.]

Special Needs
If you have learning disability or other physical impairment that may affect your ability to do the work in

this course, please let me know as soon as possible so that we can make appropriate arrangements. See the link at www.math.emich.edu/access.services.html for information about support services. In addition, international students should check the information at www.math.emich.edu/SEVIS.html concerning registration.

Academic Honesty
[I expect all students to abide by the University's code of conduct, and in particular to abide by rules concerning academic honesty. In order to assess how the class is going and what you have learned, I need to see your own work: your own words and the details of your own computations. You may work with other students or math tutors on your assignments, but you must do an independent write-up. I will give failing grades for academic dishonesty.]

Course Outline
{ A typical course outline might be as follows:

1. FUNCTIONS AND CHANGE
   Functions & Linear Functions
   Rates of Change & Applications of Functions to Economics
   Exponential Functions & Natural Logarithm
   Exponential Growth & Decay
   New Functions from old
   Proportionality, Power Functions & Polynomials
   Focus on Modeling
2. RATE OF CHANGE - THE DERIVATIVE
   Instantaneous Rate of Change/The Derivative Function
   Interpretation of The Derivative
   The Second Derivative/Marginal Cost & Revenue
3. SHORT-CUTS TO DIFFERENTIATION
   Derivative Formulas for Powers & polynomial
   Chain Rule/The Product & Quotient Rule
4. USING THE DERIVATIVE
   Local Maxima & Minima/Inflection Points
   Global Maxima & Minima/Profit, Cost & Revenue
   Average Cost
5. ACCUMULATED CHANGE / THE DEFINITE INTEGRAL
   Accumulated Change/The Definite Integral
   The Definite Integral As Area & Interpretation of Definite Integral
   The Fundamental Theorem of Calculus
6. USING THE INTEGRAL
   Average Value/Consumer & Producer Surplus
   Present & Future Value
   Relative Growth Rate (Optional)
7. PROBABILITY
   Density Functions
   Cumulative Distribution Function & Probability
   The Median & Mean

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Please submit all materials in electronic form.

Action of the Department/College

1. Department

Vote of department faculty: For ____14____ Against __0___ Abstentions ____1______

Signed by Department Head: Bette Warren 12/3/07

Department Head Date

2. College

______________________________ ____________

College Dean Date

Action of General Education Advisory Committee

Vote of General Education Committee: For ________ Against ________ Abstentions ________

______________________________ ____________

Chairperson, General Education Advisory Committee Date

Approval

______________________________ ____________

Associate Vice-President for Undergraduate Studies and Curriculum Date