1. Subject Code, Number, and Title: **MATH 311 Mathematical Problem Solving**

2. Credit Hours: ___(3) three____

2. Catalog Description

Problem solving strategies including deduction, argument by contradiction, induction, symmetry, the extreme principle, the pigeonhole principle, and invariants. These strategies are exemplified by tackling open-ended problems in the areas of algebra, combinatorics, number theory, and calculus.

Prereq: MATH211

4. This course is (check one):

   - □ an existing course with no revisions (need not go through the input system)
   - □ an existing course with revisions (attach this form to Request for Course Revision form)
   - ☑ a new course (attach this form to Request for New Course form)

5. Check the General Education requirement this course is intended to meet. If the course is to be proposed for more than one requirement, submit a separate form for each one.

   - □ Effective Communication
   - □ Quantitative Reasoning (**QR designation**)
   - ☑ Writing Intensive (**WI designation**)
   - □ Perspectives on a Diverse World
     - □ Global Awareness
     - □ U.S. Diversity
   - □ Knowledge of the Disciplines
     - □ Arts
     - □ Humanities
     - □ Science
     - □ Social Science
**Learning Beyond the Classroom (LBC designation)**

- Self and Well Being
- Community Service, Citizenship, and Leadership
- Cultural and Academic Activities and Events
- Career and Professional Development
- International and Multicultural Experience
- Undergraduate Research

6. **Rationale.** Provide a concise, clear, jargon-free explanation of why this is a General Education course and how it fits into this specific area of the program. (The rationale should explain to students why they are taking the course. It should address both why it is part of the General Education program and why it fits into the particular category.) This rationale should appear on the general course syllabus provided here and should be included in specific course syllabi given to students.

Mathematical writing skills are emphasized throughout MATH 311 for a variety of reasons. First of all, writing helps sort out the details of a mathematical argument. We frequently think we’ve solved a problem, but then, through the process of writing out the specific details of the solution, we realize there are flaws in our reasoning. In this way, writing serves as a logical filter helping us refine our arguments. Secondly, the process of writing successful and failed attempts at solving a problem creates a foundation for further exploration of related questions. That is, writing helps us mentally organize the bigger research picture, and therefore facilitates related research. Thirdly, and obviously, effective written communication is essential to growing and archiving the body of known mathematical knowledge.

The specific writing activities employed in MATH 311 are as follows. We will be presented with a battery of problem solving strategies, which we will then employ to tackle open-ended problems in a variety of areas of mathematics. A solution to such a problem consists of a written argument (a proof) explaining, using an appropriate balance of narration and mathematics, how we arrive at the conclusion. Writing such proofs is the mainstay of the course. In order to gain skill in solving open-ended problems, we will critique our own efforts by keeping a written problem-solving log in which we analyze our successes and failures and record new questions which arise during the course of our investigations. Finally, we will write an expository research paper of considerable length and depth. For these reasons, MATH 311 is a course that fulfills the **Writing Intensive** requirement for graduation.

7. Clearly and concisely explain how this course meets each of the General Education outcomes for the requirement checked in number five (all outcomes should be addressed). To do this, (a) list the General Education outcomes for the requirement and explain how the course meets each outcome; and (b) explain, in general terms, the method(s) of evaluation to be used in the course and how these methods assess the degree to which students have met the General Education outcomes for this requirement.

In the upper-level writing-intensive courses in the major, students will...

**Develop and employ successful, flexible writing and reading strategies that support sustained inquiry in a discipline.**
Students will tackle open-ended problems as opposed to straightforward exercises - and there is a difference. In working an exercise, the path to the solution is relatively straightforward and clear. In contrast, open-ended problems are rather novel and have sufficient depth so that it might not be evident where to begin at all. Typically one reads solutions to related problems and begins by tackling simpler versions of the problem and gradually works toward a general solution. Students will keep a written log of their reading, their successes, their failures, and the related questions which arise during their investigation. To summarize, the goal of the course is to integrate mathematical reading and writing into an environment of mathematical inquiry and problem solving.

**Assessment.** Students will keep a log (or journal) in which they record their various attempts, both successes and failures, to solve non-routine problems (30%). Their logs should also include any conjectures that arose from their reading and attempts at problem solving (30%). Their logs provide an opportunity for them to reflect upon their problem-solving efforts and to practice communicating mathematical ideas in writing to others. Their logs should be clearly readable by fellow students. As such, they will employ all the habits of good mathematical writing as taught in Math 211 (the prerequisite) and in this class. That is, when making logical arguments such as *direct proof, proof by contradiction, proof by induction, etc.*, students will follow the conventional formats for those arguments (20%). In addition, it is important that their writing strikes an appropriate balance between narration and mathematical notation (20%). Students learn this balance by reading and analyzing the proofs in our textbook or, for that matter, any textbook from a previous class. Students will receive much feedback on this issue in their log evaluations which will occur every two weeks. Since their log forms a record of their past efforts, it cannot be rewritten and resubmitted after evaluation. However, students should use the feedback on each two-week cycle to improve their log writing on the next cycle.

**Use writing strategies that achieve the purposes(s) for writing and address the expectations of audience(s) within a disciplinary context.**

The purpose of most mathematical writing is to explain the logical connection between statements. For example, consider the following two statements. "If *c* is the hypotenuse of a right triangle with legs *a* and *b*, then *c*² = *a*² + *b*²." The mathematician wants to provide a written logical argument of how the equation *c*² = *a*² + *b*² can be deduced from the fact that we started with a right triangle. Such a logical argument is called a *proof*. As students investigate open-ended problems, they will discover relationships which they will then articulate in the form of a written *proof*. A well-written proof follows accepted conventions, and one goal of this course is to mentor the students’ proof-writing skills so that their proofs meet the expectations of the mathematical community.

**Assessment.** Students will write proofs and submit them for evaluation almost every class period. Their proofs will be evaluated based on logical correctness (50%), balance between proper notation and narration (20%), clarity (20%), and grammatical correctness (10%). (To get a sense for this proper balance, students will be required to carefully read and analyze the proofs in our text.) After student proofs are graded and returned, they may rewrite them and resubmit them the next class period for up to 85% of the original points possible.

**Formulate research questions and employ strategies for researching and responding to those questions.**

It is very often the case that, during the investigation of a question, one discovers several new but related questions. Students will keep a log of their problem solving experiences, and in their log, they will record the new questions which arise during the course of their investigations. The goal of this course is to provide students with strategies for researching and responding to these new questions.

**Assessment.** Student's logs will be examined to see if they are formulating conjectures from their reading and explorations. (See above for a more complete description of student logs.)

In addition students will write an expository research paper of considerable length and depth. Since MATH 311 is a problem-solving course, we want students to gain some perspective on just how long and contracted the process
of solving a significant problem can be. Each student will select a particular famous mathematical problem to investigate. They will research the significance of the problem, the people involved in solving the problem, the length of time and effort required to arrive at the solution, and whatever aspects of the problem solving effort they find interesting. Their written report shall include intriguing narration and the mathematics of the problem and its solution (to the extent that a student at their level can understand). Students will be given instruction on how to find and evaluate research sources; they will compare examples of good and poor papers; they will write a research proposal for instructor approval. All of these efforts will lead to the final version of their research paper. (For a more detailed explanation of the research paper see “Understand conventions for communicating, disseminating, and interpreting information within a discipline.” below.

Use discipline-specific genres to communicate information.

The mainstay of mathematical writing is the *mathematical proof*, which is a logical explanation or argument explaining why a given statement is true. The format of the proof and the conventions followed are dependent on the type of logical argument used. Students will have ample opportunity to practice both reading and writing mathematical proofs.

Assessment. Students’ proofs will be evaluated. (See above for a more complete description of student proofs.)

Understand conventions for communicating, disseminating, and interpreting information within a discipline.

Students will be required to read statements of problems and research possible avenues of solutions in several areas of mathematics, which will provide them with practice at interpreting written mathematics. Good writing in mathematics requires the writer to strike an appropriate balance of narration and mathematical notation. In order for students to develop a sense for this balance they must read mathematics, write mathematics, and receive gentle guidance and critiques. This course provides students with all of these necessary ingredients.

Assessment. Students will write one major expository paper in the class. They will choose a mathematical topic of interest and chronicle the problem-solving efforts of others in the historical development of that topic. Their paper will address the following elements.

1. Why is your report interesting? Answer this question in the introduction of your paper in order to pique the reader’s interest.
2. What was the historical context which gave rise to the problem?
3. Who were the principle characters involved in the solution to the problem and what were the technical mathematical contributions of each? (This forms the core of your paper.)
4. What impact did the eventual solution of the problem have on subsequent mathematical developments?
5. What personal struggles did the problem solvers undergo?
6. What were the political, economic, social, and/or religious issues surrounding this problem?

Points will be assigned according to the following rubric: question 1 (5%), question 2 (10%), question 3 (40%), questions 4, 5, and 6 (10% each), sentence structure, grammar, and spelling (15%).

Students will progressively work toward the final version of their paper through the following stages.

1. They will read, analyze, and grade two sample papers against the six elements and the rubric listed above. For each of the two sample papers, they will give a written report of how they would assign points to each of the six elements and a brief explanation of why.
2. They will choose a topic for their paper. The instructor will provide a list of possible topics or the student may choose a topic of interest (with instructor approval).
3. Students will submit a proposal (at most one page) listing their topic and briefly answering each of the six questions (in one or two sentences each) listed above. In addition they will submit an annotated bibliography briefly describing how each of their sources relates to the six elements of their paper. (See the discussion below on teaching students how to find and evaluate sources.)
4. Students will submit a first draft of their paper for peer review. Students may discuss their first draft with the instructor.
5. Students will submit the final version of their paper.
(See the course syllabus for the due dates of these stages.)

Students will receive one hour of instruction on the use of appropriate mathematical research databases and on differentiating between reliable and questionable web sources. Professor Paula Storm (Science and Technology Librarian) has prepared a wonderful presentation for this purpose and has agreed to instruct our MATH 311 students. In her presentation, she will familiarize students with a large number of mathematical research databases which are described on the following link pages.


In addition, Professor Storm will present a comprehensive tutorial on recognizing whether web sources are reliable or not. Her presentation includes discussions of domain extensions (com, edu. org, etc.), how to find who authored the information, the agency that supports the site, the owner of the domain, the purpose of the site, and hoax sites. The following are samples of hoax sites.

- [webonastick.com/pi/](http://webonastick.com/pi/)
  This site provides a bogus “proof” by induction that Pi is rational. The author of this site admits that his “proof” is a joke at the very bottom of the web page. Beware of sites authored by individuals making extraordinary claims.
- [answers.yahoo.com/question/index?qid=20061107224327AAwmh2y](http://answers.yahoo.com/question/index?qid=20061107224327AAwmh2y)
  On this site, forum participants offer incorrect opinions on the probability theory involved in the game show *Deal or No Deal*. Beware of discussion forums in which individuals offer their “expertise.”

8. Attach a syllabus (1-inch margins and 10-12 pt. font). The syllabus must include the rationale from #6 above and clearly reflect the outcomes and methods of evaluation detailed in #7 above.

Please submit all materials in electronic form.

A. Action of the Department/College

1. Department

   Vote of department faculty:  
   For ___17___  
   Against ___0___  
   Abstentions ___0___

   ________________________________  
   C. J. Gardiner  
   Department Head  
   9/5/08  
   ________________________________  
   Date

2. College

   ________________________________  
   College Dean  
   ________________________________  
   Date

A. Action of General Education Advisory Committee

   Vote of General Education Committee:  
   For _____  
   Against _____  
   Abstentions ______

   ________________________________  
   Chairperson, General Education Advisory Committee  
   ________________________________  
   Date
B. Approval

__________________________________________  __________
Associate Vice-President for Undergraduate Studies and Curriculum  Date